Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page.

Name: _____

1. (10 points) **Warm Up**

   For each of the functions given below, indicate the tightest upper-bound possible.

   (a) $f(n) = \frac{1}{n^2}$?

   (b) $f(n) = (5n)^2 - 100 + 5.34$?

   (c) $f(n) = n^3$?

   (d) Is $(\frac{n}{2})^n = O(2^n)$?

   (e) What is the worst-case time complexity of accessing the ith element in a linked list of n elements?

   (f) What is the worst-case time complexity of accessing the ith element in an array of n elements?

   (g) Give a precise definition of $f = \Theta(g)$.

2. (15 points) **Heaps**

   Give an $O(1)$ algorithm for finding the third-smallest element in a balanced unique min-heap. Give your answer using pseudocode or clear English with a diagram. Prove your running time bound.

3. (20 points) **Trees**

   Prove that red-black trees are approximately balanced. (Hint: use the tree's properties to determine how different leaf depths are from each other.)

4. (10 points) **Quicksort**

   What is the running time of QUICKSORT when all elements of the input array have the same value?

5. (10 points) **Trees**

The pseudocode to find the number of nodes in a tree is as follows:

Algorithm 1 SIZE(x)

```
1: if x = nil then
2:     return 0
3: return 1 + SIZE(x.left) + SIZE(x.right)
```

What is the worst-case runtime for a call to SIZE($root$) in a tree with n nodes? What is the best-case runtime?